ACE Calculus – Mr. Bo
Unit 3A – Day 4 Notes v3

Name *Notes.*
Date:

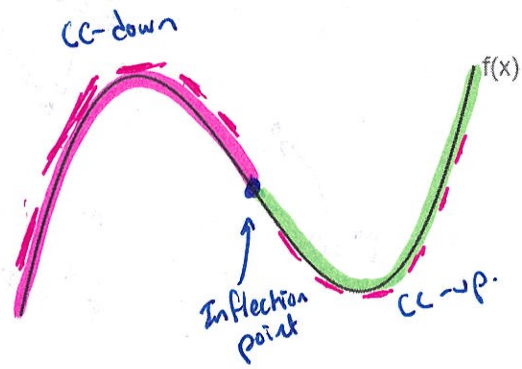
The Second Derivative Test for Relative Extrema

Concavity & Points of Inflection

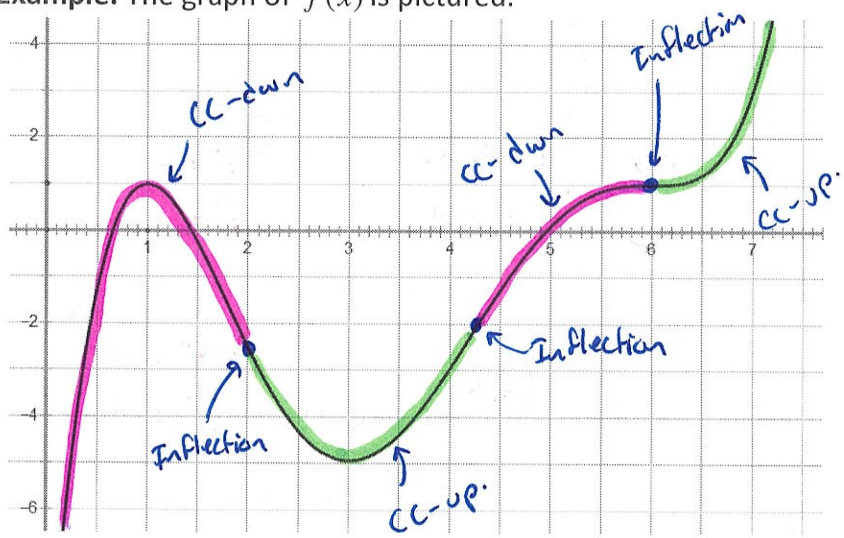
Concave Down – A curve whose slope is *decreasing*

Concave Up – A curve whose slope is *Increasing*

Inflection Point - *where concavity changes.*



Example: The graph of $f(x)$ is pictured.



a. Identify the intervals where f is cc-up.

$(2, 4.5)$ and $(6, \infty)$

b. Identify the intervals where f is cc-down.

$(-\infty, 2)$ and $(4.5, 6)$

c. Identify any points of inflection.

$x = 2$, $x = 4.5$, $x = 6$.

Concavity & the Second Derivative

Concave Down – If slope of $f(x)$ is **Decreasing**,

then $f'(x)$ is *Decreasing*

and $f''(x)$ is *negative.*

think 1st D-test →

f'' −

f' ↘

f

− −

Concave Up – If slope of $f(x)$ is **Increasing**,

then $f'(x)$ is *Increasing*

and $f''(x)$ is *positive.*

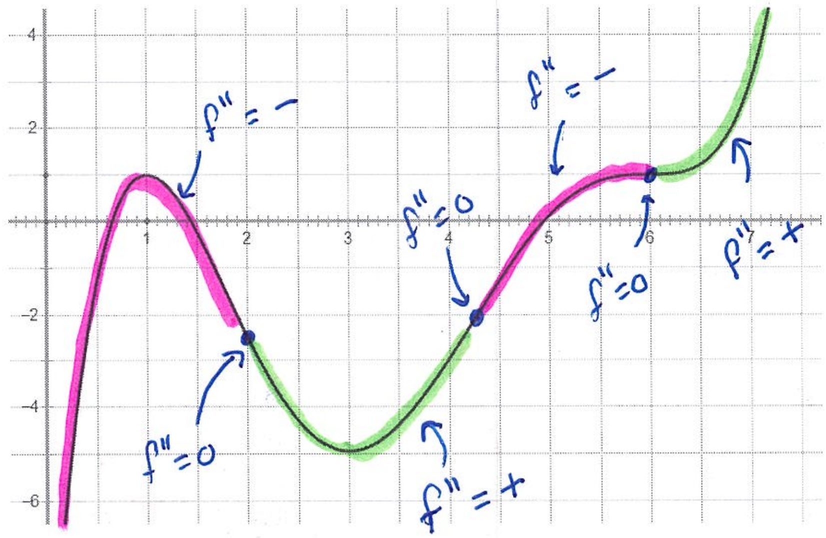
f'' +

f' ↗

f

+ +

Example: The graph of $f(x)$ is pictured.



a. Identify the intervals where $f''(x)$ is positive.

$++$ $\cup$ $(2, 4.5)$ and $(6, \infty)$

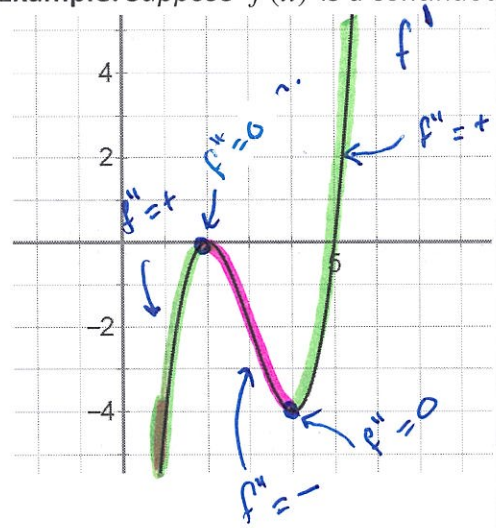
b. Identify the intervals where $f''(x)$ is negative.

$--$ $\cap$ $(-\infty, 2)$ and $(4.5, 6)$

c. Identify any points where $f''(x) = 0$.

$x = 2$ and $x = 4.5$ and $x = 6$

Example: *Suppose* $f(x)$ *is a continuous function. The graph of* $f'(x)$ *is pictured.*



a. Identify the intervals where $f(x)$ is cc-up.

$++$ $\cup$ $(-\infty, 2)$ $(4, \infty)$

$f'' = +$
$f' =$ inc.

b. Identify the intervals where $f(x)$ is cc-down.

$--$ $\cap$ $(2, 4)$

$f'' = -$
$f' =$ dec.

c. Identify any points of inflection.

$x = 2$, $x = 4$

Example: Determine the intervals where the function is cc-up/dn and any points of inflection.

$f'' = +$ or $-$ $\qquad$ $f'' = 0$

$$f(x) = x^6 - 5x^4$$

$f' = 6x^5 - 20x^3$

$f'' = 30x^4 - 60x^2 = 0$

$30x^2(x^2 - 2) = 0$

$x = 0$, $x = \pm\sqrt{2}$

possible Inflection pts.

f''	$++$		$--$		$--$		$++$
f	$\cup$	$-\sqrt{2}$	$\cap$	0	$\cap$	$\sqrt{2}$	$\cup$
		Inflection				Inflection.	

think 1st-D test.

The Second Derivative Test for Relative Extrema:

If $f(x)$ is a function such that $f'(c)=0$ and $f''(c)$ exists on an open interval containing c, then:

1. $f(c)$ is a Rel. Min. if:

$f'(c)=0$

and

$f''(c)=+$ ← f cc-up.

2. $f(c)$ is a Rel. Max. if:

$f'(c)=0$

and

$f''(c)=-$ ← f cc-down.

Example: Find the relative extrema using the 2nd derivative test.

$f(x)=x^6-5x^4$

① find C.V.'s.

$f'=6x^5-20x^3=0$

$2x^3(3x^2-10)=0$

$x=0,\ x=\pm\sqrt{10/3}$

② find f''

$f''=30x^4-60x^2$

③ test for R.E.'s

$f''(0)=0$ inconclusive.

$f''(\sqrt{10/3})=133.\overline{3}$ + + rel. min at $x=\sqrt{10/3}$

$f''(-\sqrt{10/3})=133.\overline{3}$ + + rel min at $x=-\sqrt{10/3}$

Example: Find the relative extrema using the 2nd derivative test.

$f(x)=-3x^5+5x^3$

① find C.V's.

$f'(x)=-15x^4+15x^2=0$

$-15x^2(x^2-1)=0$

$x=0,\ x=1,\ x=-1$

② find f'':

$f''=-60x^3+30x$

③ test f'' for RE's

$f''(0)=0$ inconclusive.

$f''(1)=-30$ − − rel. max.

$f''(-1)=30$ + + rel. min.