ACE Calculus I – Mr. Bo
Unit 3B – Day ~~2~~ 3 Notes

Name______________________
Date______________

Newton's Method for Approximating Zeros of a Function

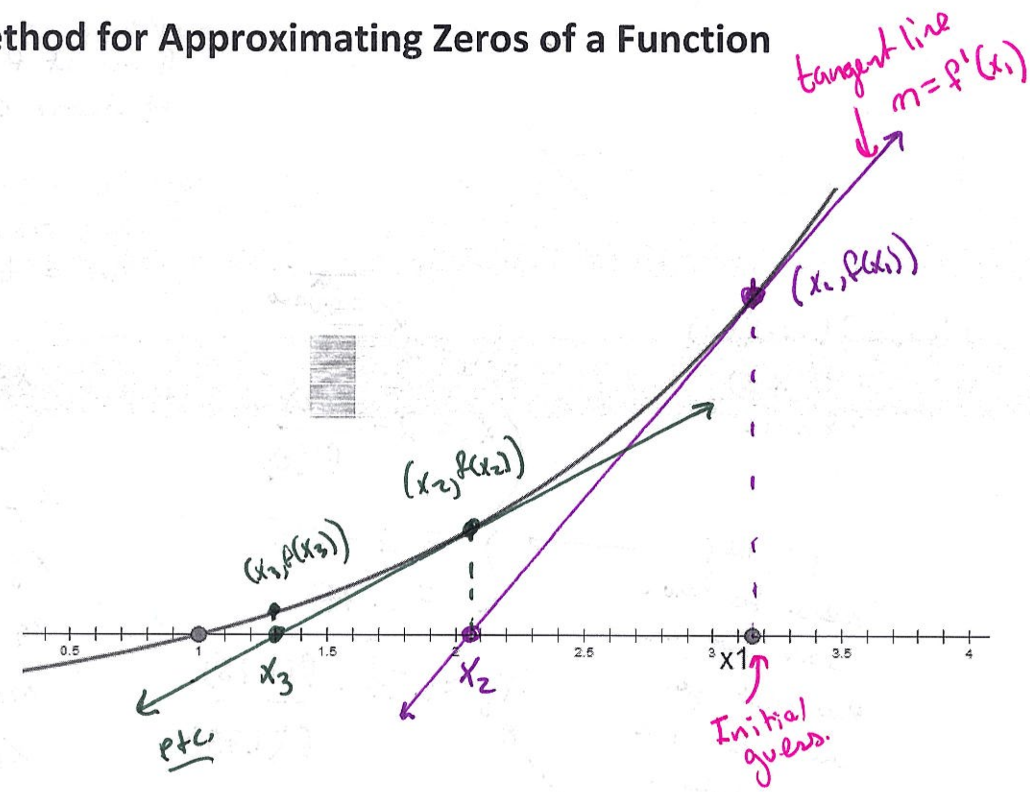
Basic Idea:

1. Find the point $(x_1, f(x_1))$ on the graph.

2. Draw the line tangent to the graph at $(x_1, f(x_1))$.

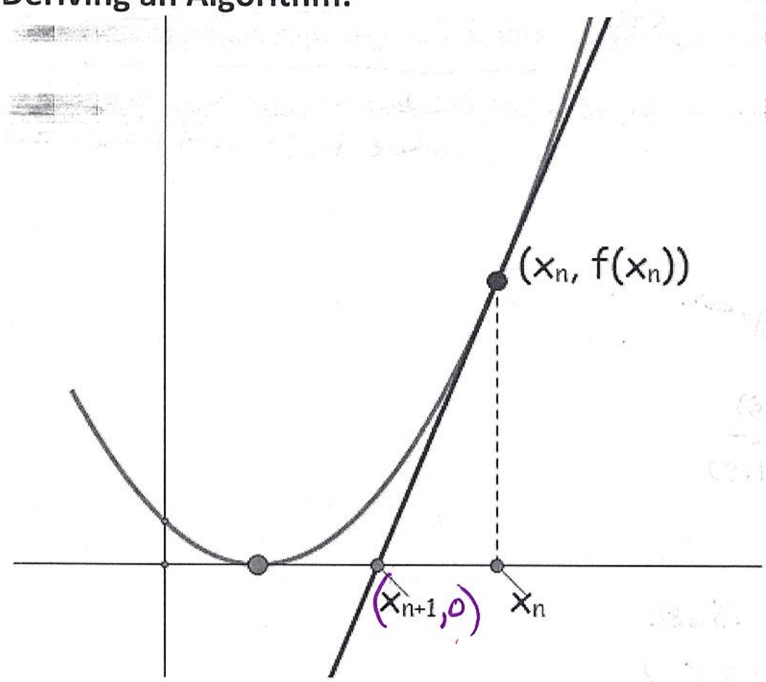
3. Locate x_2, the point where the tangent line crosses the x-axis.

4. Repeat the steps using x_2 to find x_3, x_3 to find x_4, x_4 to find x_5, etc.



Note: Newton's Method is a process (or *Algorithm*) that generates a *recursive sequence* of x values that get continually closer to the actual zero of f(x).

Deriving an Algorithm:



· use slope formula:

$$m = \frac{\Delta y}{\Delta x}$$

$$f'(x_n) = \frac{0 - f(x_n)}{x_{n+1} - x_n}$$

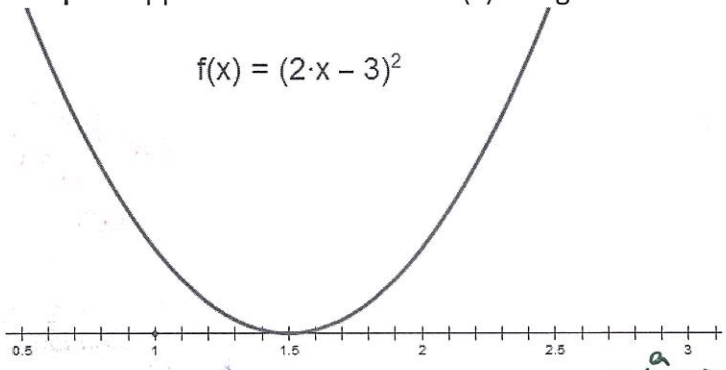
solve for this.

$$x_{n+1} - x_n = \frac{-f(x_n)}{f'(x_n)}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

recursive sequence.

Example: Approximate the zero of f(x) using Newton's Method.



Questions to consider:

How do you choose x_1?

① look at the graph.

② choose close to the zero.

When do you stop the algorithm?

when successive approximations are = to 3 decimal places.

$f'(x) = 2(2x-3)(2)$

$= 4(2x-3)$

$= 8x - 12$

$x_1 = 2 \leftarrow$ a Guess

$x_2 = 2 - \dfrac{f(2)}{f'(2)}$

$= 2 - .25$

$= 1.75$

you only need to show work for x_2 the rest you can use the calculator.

$x_3 = 1.75 - \dfrac{f(1.75)}{f'(1.75)}$

$= 1.625$

work not nec. for these 2.

$x_4 = 1.625 - \dfrac{f(1.625)}{f'(1.625)}$

$= 1.5625$ etc.

$x_5 = 1.53125$

$x_6 = 1.515625$

$x_7 = 1.5078125$

$x_8 = 1.50390625$

$x_9 = 1.501953125$

$x_{10} = \boxed{1.500}976563$

$x_{11} = \boxed{1.500}488281$

Stop!

Zero ≈ 1.500

Example: Use Newton's Method to approximate where the graphs of $f(x) = x^2$ and $g(x) = 2$ intersect.

where their difference = 0.

$f(x) - g(x) = \underbrace{x^2 - 2}$

use Newton's on this.
(we'll call it h(x))

$h(x) = x^2 - 2$

$h'(x) = 2x$

$x_1 = 1.5 \leftarrow$ a guess.

$x_2 = 1.5 - \dfrac{h(1.5)}{h'(1.5)}$

$= 1.41\overline{6}$

$x_3 = 1.414215686$

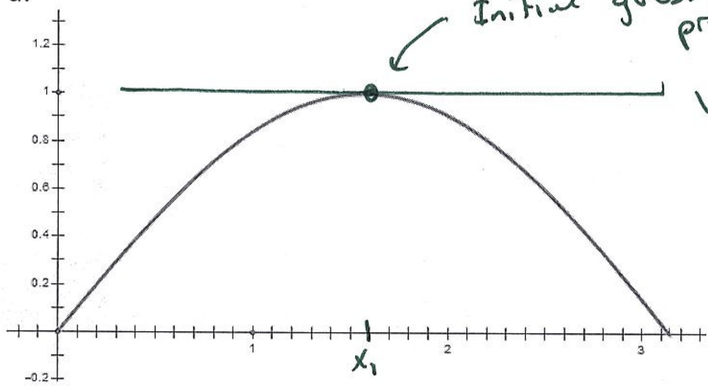
$x_4 = \underbrace{1.4142}_{\text{stop.}}13562$

f(x) and g(x) intersect at approx 1.4142

Does Newton's Method Always Work?

For each picture, describe how Newton's Method may fail to find a zero (or find the wrong zero) for the function pictured.

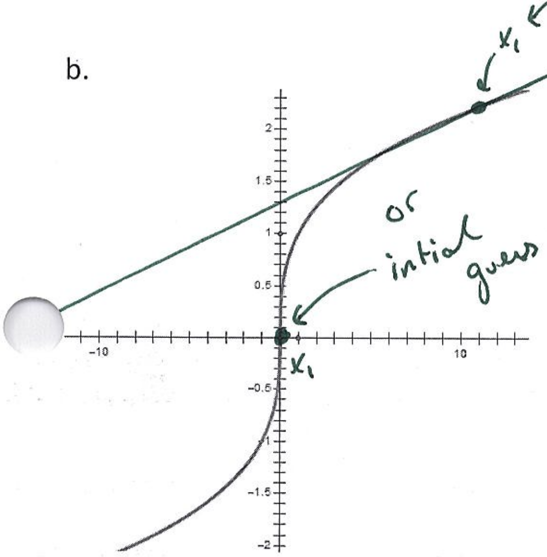
a.



Initial guess produces a horizontal tangent line.

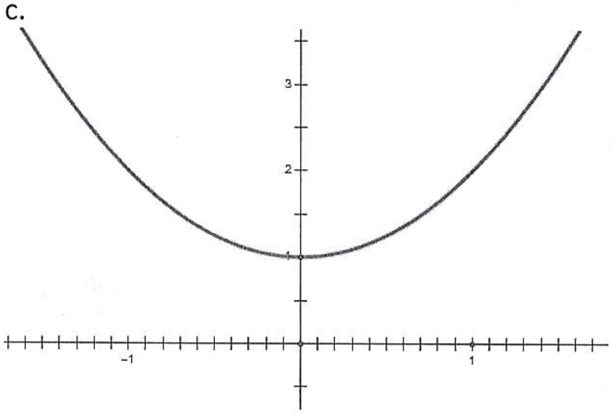
b.

Initial guess produces a sequence that does not get closer to the zero.



or initial guess produces a vert. tangent.

c.



the function has no zeros!