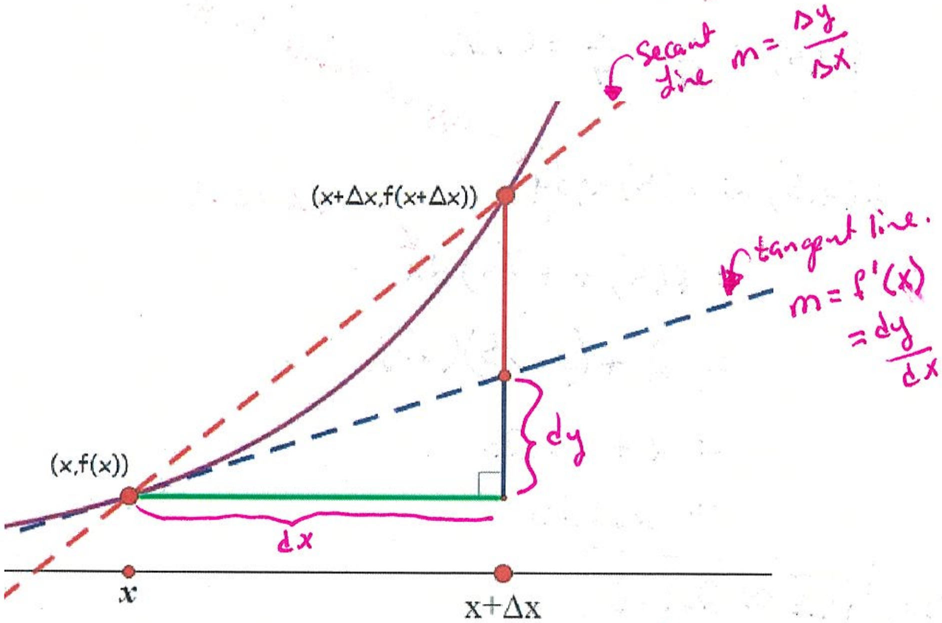
Name: notes.
Date:

Differentials



$f'(x) = \frac{dy}{dx}$

$\boxed{dy = f'(x)\,dx}$

"differentials"

Example: Find dy for each.

a. $f(x) = (x^2+1)^3$

$f'(x) = 3(x^2+1)^2(2x)$
$= 6x(x^2+1)^2$

$dy = f'(x)dx$

$\boxed{dy = 6x(x^2+1)^2 dx}$

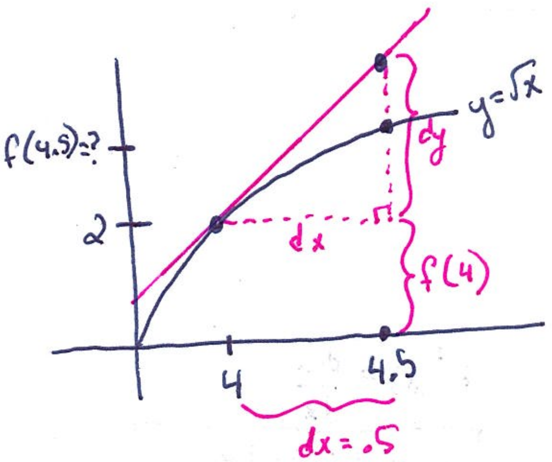
b. $f(x) = 2\sin(3x)$

$f'(x) = 2\cos(3x)\cdot 3$
$= 6\cos(3x)$

$dy = f'(x)dx$

$\boxed{dy = 6\cos(3x)dx}$

Example: Suppose $f(x) = \sqrt{x}$. Use differentials to approximate $f(4.5)$.



$f'(x) = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4} = 0.25$

$f(4.5) \approx f(4) + dy$ ← approx change in y.

$= f(4) + f'(4)\,dx$ ← $dx = 0.5$

$= 2 + (0.25)(.5)$

$= 2 + 0.125$

$= 2.125$

So, $\boxed{\sqrt{4.5} \approx 2.125.}$

this is an over estimate.

Actual Values:

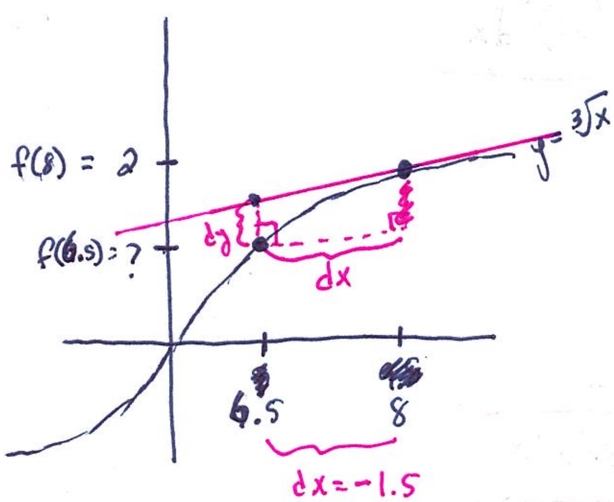
$\Delta y = f(4.5) - f(4)$

$= \sqrt{4.5} - \sqrt{4}$

$= 0.121320$

$f(4.5) = 2.1213$

Example: Suppose $f(x) = \sqrt[3]{x}$. Use differentials to approximate $f(6.5)$



$f'(x) = \frac{1}{3}x^{-2/3}$

$= \frac{1}{3\sqrt[3]{x^2}}$

$f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3 \cdot 4} = \frac{1}{12} = 0.08\overline{3}$

$f(6.5) \approx f(8) + dy$

$= f(8) + f'(8)\,dx$ ← $dx = -1.5$

$= 2 + (0.08\overline{3})(-1.5)$

$= 2 - 0.125$

$= 1.875$

So, $\boxed{\sqrt[3]{6.5} \approx 1.875}$

this is an over estimate.

Actual values

$\Delta y = f(6.5) - f(8)$

$= -0.1337$

$f(6.5) = 1.8663.$

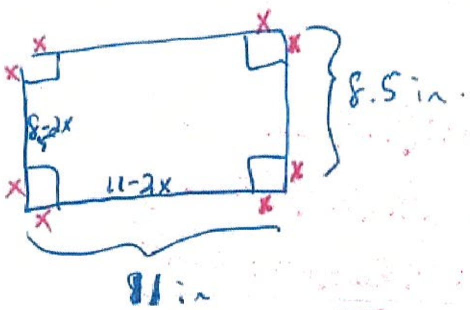
Differentials & Propagated Error

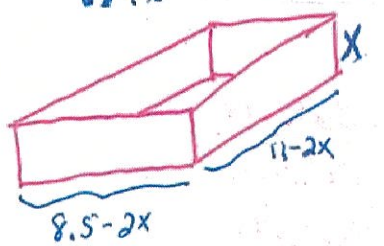
Measurement Error: Approximate error made when measuring = dx

Propagated Error: The uncertainty in a calculated measure due to measuring error = dy

Relative Error = $\frac{dy}{y}$ **Percent Error** = $\frac{dy}{y} \cdot 100$

Example: In a previous lesson, it was determined that squares of 1.6 inches cut from the corners of a single piece of 8.5 inch by 11-inch piece of cardboard would produce an open top box of largest volume. Suppose we wanted to make this box, but in measuring the sides, we could only be accurate up to ±1/16 of an inch. Determine the propagated error and percent error for the volume of the resulting box.





$$V = x(8.5-2x)(11-2x)$$
$$= 93.5x - 39x^2 + 4x^3$$
$$V' = 93.5 - 78x + 12x^2$$

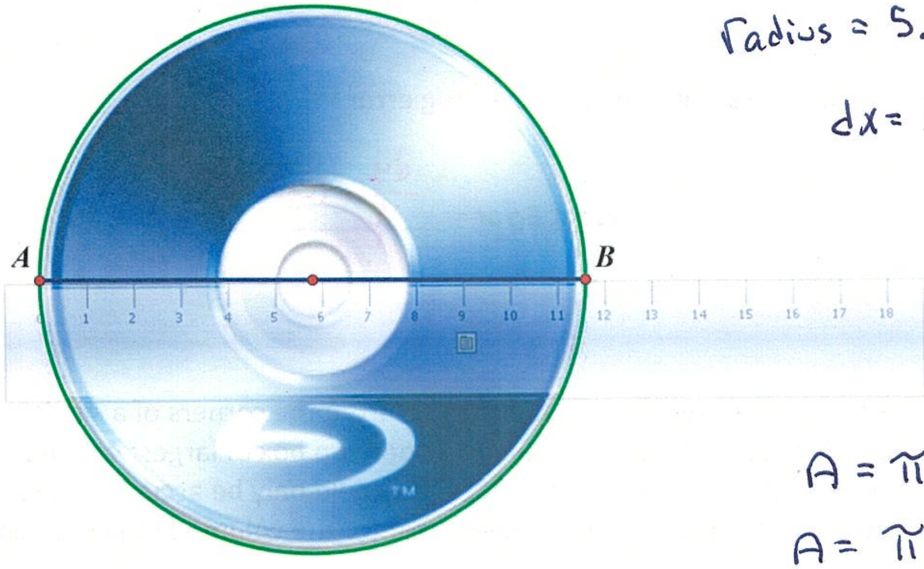
$dx = \pm 1/16 = \pm 0.0625$ inches. ← measurement error

$$dy = V'(1.6)\,dx$$
$$= (-0.58)(\pm 0.0625)$$
$$\boxed{= \pm 0.03625 \text{ in}^3}$$ ← propagated error.

$$\frac{dy}{y} = \frac{\pm 0.03625}{V(1.6)}$$
$$= \frac{\pm 0.03625}{66.144} = -0.000548$$

$$\boxed{\% \text{ error} = 0.0548\%}$$

Example: The radius of a Blue-Ray disk is measured for calculating its approximate area. Using the ruler provided in the picture, determine the propagated error and the percent error of the area of the disk.



radius = 5.8 cm, ± 0.1 cm.

$dx = \pm 0.1$ cm.

$A = \pi r^2$

$A = \pi x^2$

$A' = 2\pi x$

$dy = A'(5.8)\,dx$

$= 2\pi(5.8)(\pm 0.1)$

propagated. $\rightarrow$ $\boxed{= \pm 3.644 \text{ cm}^2}$

$\frac{dy}{y} = \frac{\pm 3.644}{A(5.8)}$

$= \frac{3.644}{105.683}$

$= 0.03448$

$\boxed{\%\ \text{error} = 3.448\ \%}$