

CONSECUTIVE INTEGER WORD PROBLEMS

One of the ways we can practice our ability to work with algebraic expressions and equations is to play around with problems that involve **consecutive integers**. Make sure you know what the integers are:

THE INTEGERS AND CONSECUTIVE INTEGERS

The **integers** are the subset of the **real numbers**: $\{\dots -4, -3, -2, -1, 0, 1, 2, 3, \dots\}$ (so positive and negative whole numbers).

Consecutive integers are any list of integers (however long) that are separated by only 1 unit. Such as:

$$1, 2, 3 \text{ or } 5, 6, 7, 8 \text{ or } -4, -3, -2 \text{ or } -10, -9, -8, -7, -6$$

Consecutive Evens

$$4, 6, 8 \text{ or } -8, -6, -4, -2 \text{ or } 14, 16$$

Consecutive Odds

$$7, 9, 11 \text{ or } -5, -3, -1, 1 \text{ or } -9, -7, -5$$

Exercise #1: Let's work with just two consecutive integers first. Say we have two consecutive integers whose sum is eleven less than three times the smaller integer.

(a) It is important to play around with this problem numerically. So, try a variety of combinations and see if you can find the correct pair of consecutive integers. Be sure to show your calculations.

$$1 + 2 = 3(1) - 11$$

$$3 = 3 - 11$$

$$3 \neq -8$$

$$5 + 6 = 3(5) - 11$$

$$11 = 15 - 11$$

$$11 \neq 4$$

$$10 + 11 = 3(10) - 11$$

$$21 = 30 - 11$$

$$21 \neq 19$$

(b) Now, carefully set up let statements that give expressions for our two consecutive integers. Using these expressions, set up an equation that allows you to find them and solve the equation.

let $x = 1^{\text{st}}$ consecutive integer (11)

$x + 1 = 2^{\text{nd}}$ consecutive integer (12)

$$x + x + 1 = 3x - 11$$

$$\begin{array}{r} 2x + 1 = 3x - 11 \\ -2x \quad -2x \\ \hline \end{array}$$

$$1 = x - 11$$

$$\begin{array}{r} + 11 \quad + 11 \\ \hline \end{array}$$

$$12 = x$$

$$\text{ck: } 11 + 12 = 23 \checkmark$$

$$3(11) - 11 =$$

$$33 - 11 = 23 \checkmark$$

The two consecutive integers are 11 and 12.

Exercise #2: I'm thinking of three consecutive odd integers. When I add the larger two the result is nine less than three times the smallest of them. What are the three consecutive odd integers?

let $x = 1^{\text{st}}$ cons. odd int (15) $x+4 + x+2 = 3x-9$ CK: $17+19 = 36$
 $x+2 = 2^{\text{nd}}$ cons odd int (17) $2x+6 = 3x-9$ $3(15)-9 = 36 \checkmark$
 $x+4 = 3^{\text{rd}}$ cons odd int (19) $-2x \quad -2x$

$$\begin{array}{r} 2x+6 = 3x-9 \\ -2x \quad -2x \\ \hline 6 = x-9 \\ +9 \quad +9 \\ \hline 15 = x \end{array}$$

The three cons. odd int are 15, 17, and 19

Exercise #3: Three consecutive even integers have the property that when the difference between the first and twice the second is found, the result is eight more than the third. Find the three consecutive even integers.

let $x = 1^{\text{st}}$ cons even int (-8) $x - 2(x+2) = 8 + x + 4$ CK: $-8 - 2(-6) = 4$
 $x+2 = 2^{\text{nd}}$ cons even int (-6) $x - 2x - 4 = x + 12$ $-4 + 8 = 4 \checkmark$
 $x+4 = 3^{\text{rd}}$ cons even int (-4)

$$\begin{array}{r} -x - 4 = x + 12 \\ +x \quad +x \\ \hline -4 = 2x + 12 \\ -12 \quad -12 \\ \hline -16 = 2x \\ \frac{-16}{2} = \frac{2x}{2} \\ -8 = x \end{array}$$

The three consecutive even integers are -8, -6, & -4.

Exercise #4: The sum of four consecutive integers is -18. What are the four integers?

let $x = 1^{\text{st}}$ cons int (-6) $x+x+1+x+2+x+3 = -18$ CK: -6
 $x+1 = 2^{\text{nd}}$ cons int (-5) $4x+6 = -18$ -5
 $x+2 = 3^{\text{rd}}$ cons int (-4) $-6 \quad -6$ -4
 $x+3 = 4^{\text{th}}$ cons int (-3)

$$\begin{array}{r} 4x+6 = -18 \\ -6 \quad -6 \\ \hline 4x = -24 \\ x = -6 \end{array}$$

$\begin{array}{r} + -3 \\ -18 \checkmark \end{array}$

The 4 cons int are -6, -5, -4, & -3.