Geometry R – Mrs. Cowen
Unit 2B – Day 4 Notes

Name______________________________
Date________________

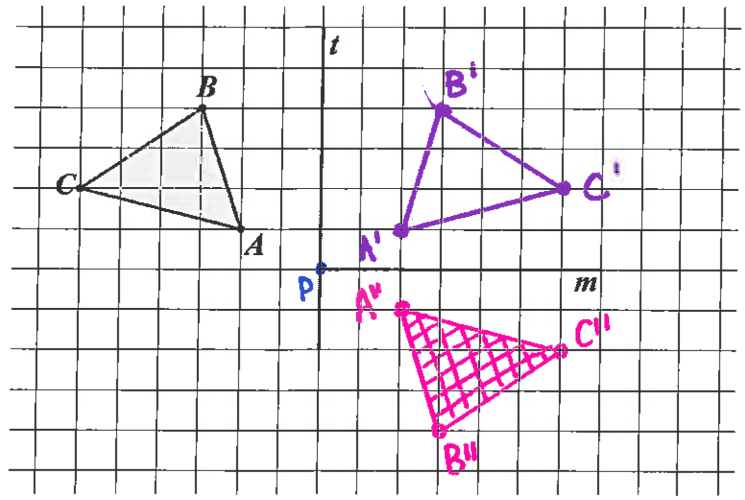
Sequences of Rigid Motions

Composition - A sequence of 2 or more transformations applied to a shape.

Isometry – A sequence of rigid motions that result in a congruent figure.

1a.

$$\underbrace{r_{line\,m}}_{\boxed{\text{do second}}} \circ \underbrace{r_{line\,t}}_{\boxed{\text{do first}}}$$



1b. Is the composition in part a an **Isometry**? Explain your reasoning.

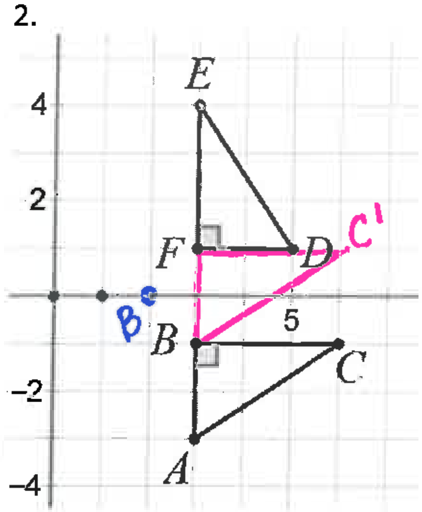
Yes, $\Delta A"B"C"$ is congruent to ΔABC.
Reflection is a rigid motion & preserves distance.

1c. In many cases, an *Isometry* can be simplified into a single rigid motion. Precisely describe a **single** rigid motion that would map ΔABC directly onto $\Delta A"B"C"$?

Rotate ΔABC 180° about the intersection of lines t & m.

or Rotate ΔABC counterclockwise around point (P) ← where t & m intersect

2.



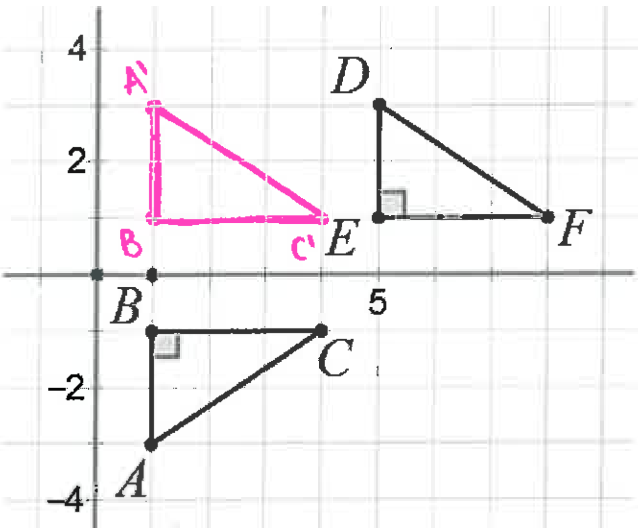
a. Describe a precise sequence of 2 or more rigid motions which would map ΔABC onto ΔDFE.

- translate ΔABC 2 units up. So that B maps to F to form $\Delta FBC'$
- Rotate $\Delta FBC'$ 90° counterclockwise so C' maps to E.

b. Precisely describe a single rigid motion that would also map ΔABC onto ΔDFE.

Rotate ΔABC 90° counter clockwise around point P.

3.



a. Precisely describe a sequence of rigid motions which would map ΔABC onto ΔDEF.

① Reflect ΔABC in the x-axis to form $\Delta A'B'C'$

② Translate $\Delta A'B'C'$ 4 units right to form ΔDEF.

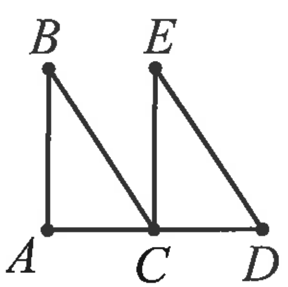
b. Is it possible to describe a single rigid motion that would map ΔABC onto ΔDEF? If so, describe precisely the transformation or if not, explain why not.

NO. The composition of a reflection and a translation can't be represented by a single rigid motion.

Using Rigid Motion to Justify Congruency

Theorem: If 2 figures in a plane are congruent, then there will always be a sequence of rigid motions that will map one figure onto the other.

4.



*we will assume we have no way to measure in the picture.

a. Precisely describe a single rigid motion that would map A onto C.

Translate point A along vector $\vec{AC}$ so that A maps to C.

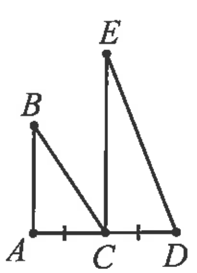
b. Under this same rigid motion would C be mapped to D? Why or why not? Explain your reasoning.

It might not. We don't know if the distance from C to D is the same as from A to C.

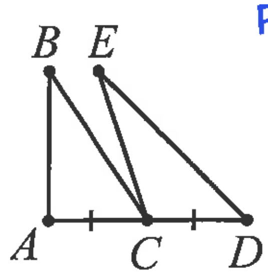
c. In order for C to be mapped to D under this rigid motion, what additional information must be known about the picture?

we must know that $\overline{AC} \cong \overline{CD}$

d. Now suppose $\overline{AC} \cong \overline{CD}$, so that C maps to D under this rigid motion. What additional information must be known for B to map to E? *Use these two pictures to help guide your reasoning.*



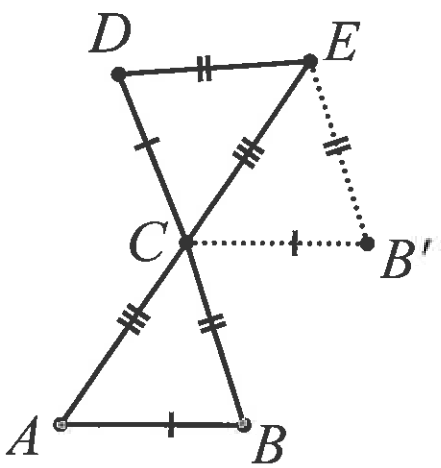
pic 1



pic 2

For B to map to E
- we would need $\overline{AB} \cong \overline{CE}$ (eliminate the possibility of pic 1)
- we would need to know $\angle BAC \cong \angle ECD$ (eliminate the possibility of pic 2)

6. Precisely describe a sequence of rigid motions that maps ΔABC onto ΔCDE.



The 1st rigid motion maps ΔABC to $\Delta CB'E$.

a. Precisely describe the 1st Rigid Motion:

Translate ΔABC along $\overrightarrow{AC}$ so that $A \to C$ and $C \to E$

b. How do you know that C maps to E?

Since $\overline{AC} \cong \overline{CE}$ and A and C will move the same distance.

The 2nd rigid motion maps $\Delta CB'E$ to ΔCDE.

c. Precisely describe the 2nd Rigid Motion:

Reflect $\Delta CB'E$ over $\overline{CE}$ so that B' maps to D.

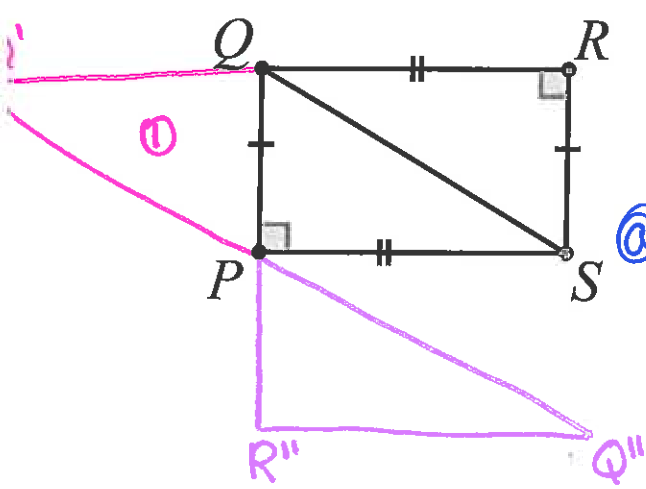
d. How do you know that B' maps to D?

Since angle measure and segment length are preserved under translation and reflection (both rigid motions) B' will map to D.

6e. How do you know that $\angle B \cong \angle D$?

Translation and Reflection are rigid motions preserving angle measure.

7.



a. Precisely describe a sequence of rigid motions that maps ΔPQS onto ΔRSQ. *Sketch the resulting triangle for each rigid motion in the sequence.*

b. Explain how the sequence of rigid motions make $\angle RQS \cong \angle PSQ$.

ⓐ ① Translate ΔQRS along $\overrightarrow{RQ}$ so that R maps to Q and S maps to P to form $\Delta Q'QP$.

② Rotate $\Delta Q'QP$ clockwise around P 180° to form $\Delta Q''R''P$

③ Translate $\Delta Q''R''P$ along vector $\overrightarrow{PQ}$ so $\Delta Q''R''P$ maps to ΔSPQ

ⓑ $\angle RQS \cong \angle PSQ$ because translations and rotations are rigid motions, preserving angle measure.