

### Finding Distance & Midpoint

The distance between two points on a grid can be found using the **Pythagorean Theorem**:

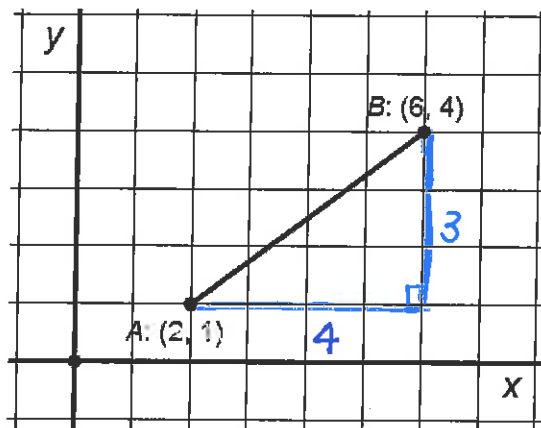
$$c^2 = a^2 + b^2$$

1. Use the **Pythagorean Theorem** to find AB.

(Note: AB means "the distance between points A and B")

Show your work here:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + 4^2 &= (AB)^2 \\ 9 + 16 &= (AB)^2 \\ \sqrt{25} &= \sqrt{(AB)^2} \\ \boxed{5} &= AB \end{aligned}$$

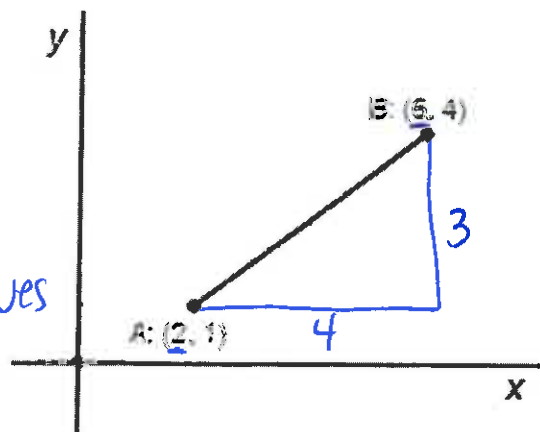


2. In the previous example you probably counted the boxes on the grid to find the lengths of the rise and the run of segment AB.

Suppose the picture instead was given without a grid.

Explain how the lengths of the rise and run can be found in this situation.

Take the difference between the 2 x-values & the 2 y-values and then use the Pythagorean Theorem.



3. Now, suppose the previous example was given without a picture, such as:

"Find the distance between points A(2,1) and B(6,4)."

Explain the strategy you would use to solve this problem.

(Note: Your strategy should "not" include using/drawing a picture)

$$\begin{array}{l} \Delta X = 4 \\ \Delta Y = 3 \end{array} \quad \left. \vphantom{\begin{array}{l} \Delta X = 4 \\ \Delta Y = 3 \end{array}} \right\} \text{ Plug into Pythag. Thm.}$$

**The Distance Formula:** The distance between two points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  can be found using the following formula:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

4. This formula is derived directly from the Pythagorean Theorem. Using your results from #1-3, explain how this could be the case.

$$\Delta X = 4$$

$$\Delta Y = 3$$

$$c^2 = a^2 + b^2$$

$$(AB)^2 = (4)^2 + (3)^2$$

$$(AB)^2 = 16 + 9$$

$$\sqrt{(AB)^2} = \sqrt{25}$$

$$AB = 5$$

5. Segment PQ has endpoint P(-5, 6) and Q(2, -4). Use the distance formula to find PQ, to the nearest tenth.

$$PQ = \sqrt{(2 - (-5))^2 + (-4 - 6)^2}$$

$$PQ = \sqrt{7^2 + (-10)^2}$$

$$PQ = \sqrt{49 + 100}$$

$$PQ = \sqrt{149}$$

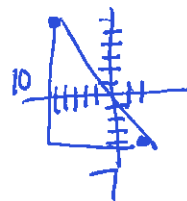
$$PQ = 12.2$$

$$(PQ)^2 = 7^2 + 10^2$$

$$(PQ)^2 = 49 + 100$$

$$\sqrt{(PQ)^2} = \sqrt{149}$$

$$PQ = 12.2$$



## The Midpoint Formula

6. Is the following statement true or false? Explain why and support your claim with 2 examples.

*"The average of any two numbers will always be a value that is half way between the two numbers."*

True

2 and 6	}	12 and 43	}	8 and 12
$\frac{2+6}{2} = \frac{8}{2} = 4$		$\frac{12+43}{2} = \frac{55}{2} = 27.5$		$\frac{8+12}{2} = \frac{20}{2} = 10$

7. If Segment AB has endpoints  $A(x_1, y_1)$  and  $B(x_2, y_2)$  then the midpoint, M, of segment AB is:

$$M \left( \overset{\text{Avg } x\text{'s}}{\frac{x_1 + x_2}{2}}, \overset{\text{Avg } y\text{'s}}{\frac{y_1 + y_2}{2}} \right)$$

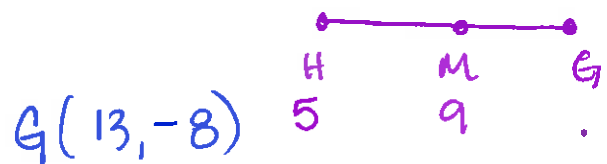
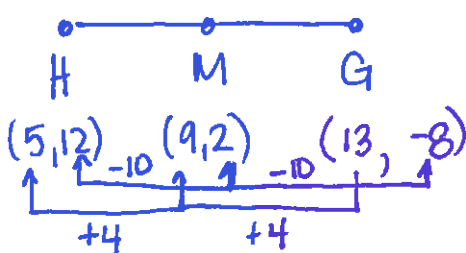
Explain how the claim in #6 supports or justifies this formula.

The formula averages the x's and averages the y's  
Therefore the resulting point is half way between the 2 given points.

8. Use the **Midpoint Formula** to find the midpoint of  $\overline{AB}$ , if  $A(2,1)$  and  $B(6,4)$ .

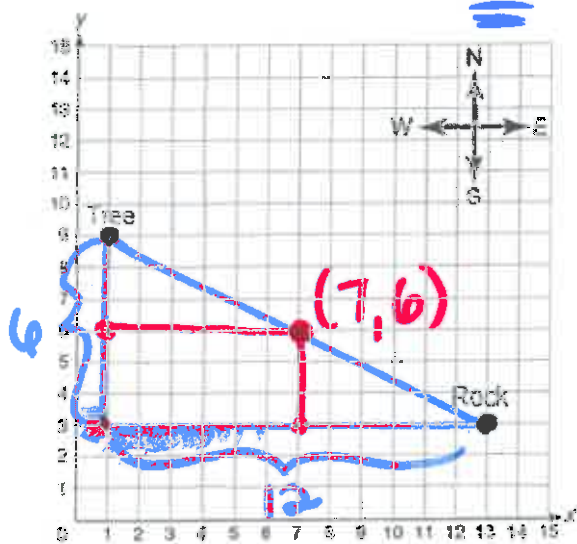
$$\begin{aligned} M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{2 + 6}{2}, \frac{1 + 4}{2} \right) \\ &= \left( \frac{8}{2}, \frac{5}{2} \right) \\ M &= (4, 2.5) \end{aligned}$$

9. Line Segment HG has endpoint  $H(5, 12)$  and midpoint  $M(9, 2)$ . Find and state the coordinates of G, the other endpoint.



10. Sometimes it is convenient to count the rise and run on a grid when finding a midpoint.

a. Use the grid to find a point that is  $\frac{1}{2}$  of the way between the tree and rock.

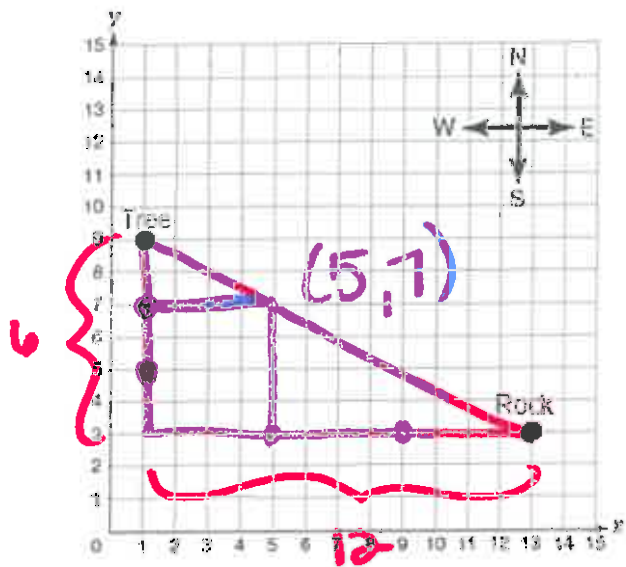


$$\frac{6}{2} = 3$$

$$\frac{12}{2} = 6$$

$$\text{Midpoint} = (7, 6)$$

b. Adapt your strategy from the last example to find a point that is  $\frac{1}{3}$  of the way between the tree and rock.



$$\frac{6}{3} = 2$$

$$\frac{12}{3} = 4$$

$(5, 7)$  is  $\frac{1}{3}$  of the way between the Tree & the Rock.