Algebra 1R
Unit 2 ~ Day 4 Notes

Name ______________________

## LITERAL EQUATIONS

At this point we should feel very competent solving linear equations. In many situations, we might even solve equations when there are **no actual numbers given**. Let's take a look at what we mean in Exercise #1.

***Exercise* #1:** Solve each of the following problems for the value of $x$. In (b), write your answer in terms of the unspecified constants $a$, $b$; and $c$.

(a) $5x+3=33$

$-3 \quad -3$

$\frac{5x}{5}=\frac{30}{5}$

$x=6$

(b) $ax+b=c$

$-b \quad -b$

$\frac{ax}{a}=\frac{c-b}{a}$

$x=\frac{c-b}{a}$

The rules for solving linear equations (and all equations) don't depend on whether the constants in the problem are specified or not. The biggest difference in #1 between (a) and (b) is that in (b) you have to leave the results of the intermediate calculation undone.

***Exercise* #2:** Solve the following two equations. In letter (b), leave your answer in terms of the constants $a$, $b$, $c$ and $d$.

(a) $\frac{x+5}{2}-7=3$

$+7 \quad +7$

$2\cdot\frac{x+5}{2}=10\cdot 2$

$x+5=20$

$-5 \quad -5$

$x=15$

(b) $\frac{x+a}{b}-c=d$

$+c \quad +c$

$b\cdot\frac{x+a}{b}=(d+c)\,b$

$x+a=b(d+c)$

$-a \quad -a$

$x=b(d+c)-a$

Of course, we can have numbers we known (specified constants) thrown into the mix. The most important thing is to know when we can combine and produce a result and when we can't.

***Exercise* #3:** When $2(x-h)+k=8$ is solved for $x$ in terms of $h$ and $k$, its solution is which of the following? Show the algebraic manipulations you used to get your answer.

(1) $4+h-k$

(2) $h+4-\frac{k}{2}$

(3) $k-\frac{h}{2}+8$

(4) $4-h+k$

$2(x-h)+k=8$

$-k \quad -k$

$2(x-h)=8-k$

$2x-2h=8-k$

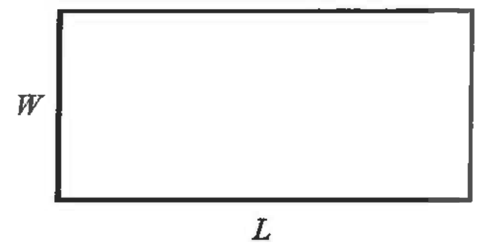
$+2h \quad +2h$

$\frac{2x}{2}=\frac{8}{2}-\frac{k}{2}+\frac{2h}{2}$

$x=4-\frac{k}{2}+h$

Many times this technique is used when we want to **rearrange** a **formula** to solve for a **quantity of interest**.

***Exercise #4:*** For a rectangle, the **perimeter**, $P$, can be found if the two dimensions of length, $L$, and width, $W$, are known.




(a) If a rectangle has a length of 12 inches and a width of 5 inches, what is the value of its perimeter? Include units.

12+5+12+5 = 34inches = perimeter

(b) Write a formula for the perimeter, $P$, in terms of $L$ and $W$.

$P = 2l + 2W$

(c) Rearrange this formula so that it "solves" for the length, $L$. Determine the value of $L$ when $P = 20$ and $W = 4$.

$P = 2L + 2W$

$20 = 2L + 2(4)$

$20 = 2L + 8$

$-8 \quad -8$

$12 = 2L \implies 6 = L$

There is one last complication that we need to look at that is often challenging for students at all levels. Let's take a look at this in the next problem.

***Exercise #5:*** Consider the equation $ax + b = cx + d$. We'd like to solve this equation for $x$. Let's start with the situation where we know the values of $a$, $b$, $c$ and $d$.

(a) Solve: $8x + 1 = 5x + 22$

$-5x \quad -5x$

$3x + 1 = 22$

$-1 \quad -1$

$\frac{3x}{3} = \frac{21}{3}$

$x = 7$

(b) Now solve: $ax + b = cx + d$

$-cx \quad -cx$

$ax - cx = b + d$

$\frac{x(a-c)}{a-c} = \frac{b+d}{a-c}$

$x = \frac{b+d}{a-c}$

***Exercise #6:*** Which of the following solves the equation $ax - k = 3(x + h)$ for $x$ in terms of $a$, $k$, and $h$. Show the manipulations to find your answer.

(1) $\dfrac{3h+k}{a-3}$ (3) $\dfrac{k+3h}{a+3}$

(2) $\dfrac{3a+k}{h-1}$ (4) $\dfrac{h+3}{a+k}$

$ax - k = 3(x+h)$

$ax - k = 3x + 3h$

$+k \quad +k$

$ax = 3x + 3h + k$

$-3x \quad -3x$

$ax - 3x = 3h + k$

$x(a-3) = 3h + k$

$\rightarrow x = \frac{3h+k}{a-3}$

Another way to determine the solution set of an inequality is to solve it algebraically. To solve an inequality means to determine the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation: You want to isolate the variable on one side of the inequality symbol.

In order to earn two $55 gift cards, Alan's total sales, $f(b)$, needs to be at least $1100. You can set up an inequality and solve it to determine the number of boxes Alan needs to sell.

$$f(b) \geq 1100$$

$$3.75b + 25 \geq 1100$$

Solve the inequality in the same way you would solve an equation.

$$3.75b + 25 \geq 1100$$

$$3.75b + 25 - 25 \geq 1100 - 25$$

$$3.75b \geq 1075$$

$$\frac{3.75b}{3.75} \geq \frac{1075}{3.75}$$

$$b \geq 286.66\ldots$$

Alan needs to sell at least 287 boxes of popcorn to earn two $55 gift cards.

Why was the answer rounded to 287?

You cant sell partial boxes of popcorn

Write and solve an inequality for each.

a) What is the greatest number of boxes Alan could sell and still not have enough to earn the Cyclone Sprayer?

$$600 \geq 3.75b + 25$$
$$-25 \qquad -25$$
$$575 \geq 3.75b$$
$$153.\bar{3} \geq b$$

153 boxes of popcorn

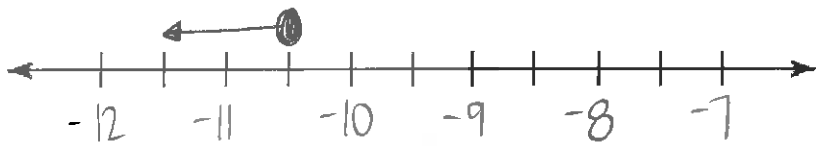
b) At least how many boxes would Alan have to sell to be able to choose his own prize?

$$3.75b + 25 \geq 1500$$
$$-25 \qquad -25$$
$$3.75b \geq 1475$$
$$b \geq 393.3$$

Alan would have to sell at least 394 boxes of popcorn to choose his own prize.

Solve each inequality and then graph the solution on the number line.

**a.** $-\frac{2}{3}x \geq 7$

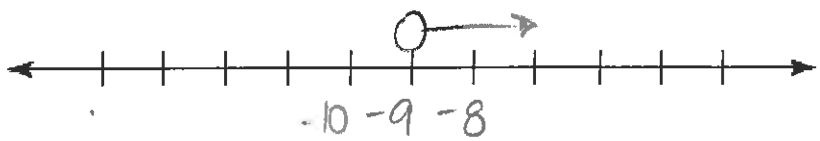



$$-2x \geq 21$$

$$\frac{-2x}{-2} \geq \frac{21}{-2}$$

$$x \leq -10.5$$

**b.** $32 > 23 - x$




$$-23 \quad -23$$

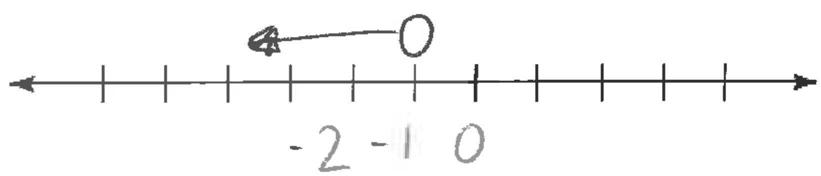
$$9 > -x$$

$$\frac{9}{-1} \quad \frac{-x}{-1}$$

$$-9 < x$$

$$x > -9$$

**c.** $2(x + 6) < 10$




$$2x + 12 < 10$$

$$-12 \quad -12$$

$$\frac{2x}{2} < \frac{-2}{2}$$

$$x < -1$$