

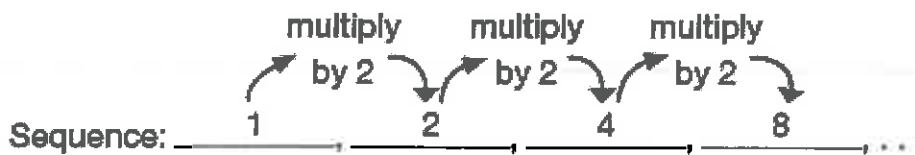
**Geometric Sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. You multiply each term by a constant to determine the next term. This is called the common ratio.



Consider the sequence shown.

1, 2, 4, 8, ...

The pattern is to multiply each term by the same number, 2, to determine the next term.



This sequence is geometric and the common ratio  $r$  is 2.

Examples: Find the "r" value.

1) 1, 2, 4, 8, 16  
 $\times 2 \quad \times 2 \quad \times 2$

$r = 2$

2) 81, 27, 9, 3, 1,  $\frac{1}{3}$   
 $\div 3 \quad \div 3 \quad \div 3$   
 $r = \frac{1}{3}$

\* can't use divide. when dividing use a fraction.

3) Consider the sequence shown.

270, 90, 30, 10, ...

Devon says that he can determine each term of this sequence by multiplying each term by  $\frac{1}{3}$ , so the common ratio is  $\frac{1}{3}$ . Chase says that he can determine each term of this sequence by dividing each term by 3, so the common ratio is 3. Who is correct? Explain your reasoning.

Devon is correct. The next term is determined by multiplying by  $\frac{1}{3}$ . The common ratio represents the number you must multiply by to find the next term not divide by as chase is doing.

Write the first five terms of each sequence described.

- 1) The first term of the sequence is 5 and the common ratio is 6.

5, 30, 180, 1080, 6480

- 2) The first term of the sequence is 1024 and the common ratio  $-1/4$ .

1024, -256, 64, -16, 4

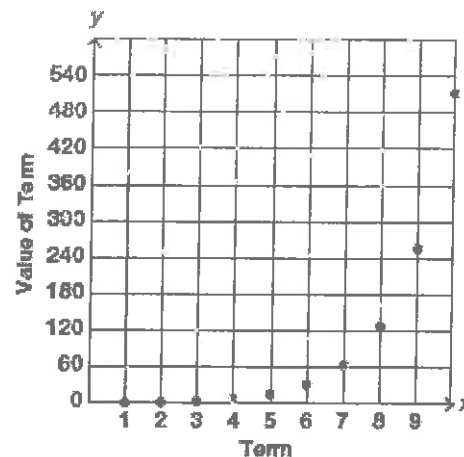
Graphs of Geometric Sequences are exponential.

Consider sequence  $g$  represented by the explicit formula shown.

$$g_1 = 1$$
$$g_n = 2^{n-1}$$

- a. Create a table of values using the first ten terms of sequence  $g$ .

Term Number ( $n$ )	Term Value
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128
9	256
10	512



Function Family  
- exponential  
- discrete

General Rule	Example
A lowercase letter is used to name a sequence.	$a$
The first term, or initial term, is referred to as $a_1$ .	$a_1 = 125$
The remaining terms are named according to the term number.	$a_2 = 143,$ $a_3 = 161, \dots$
A general term of the sequence is referred to as $a_n$ , also known as the $n$ th term, where $n$ represents the <i>index</i> .	$a_n$
The term previous to $a_n$ is referred to as $a_{n-1}$ .	$a_{n-1}$

A **recursive formula** expresses each new term of a sequence based on the preceding term in the sequence.

$$\begin{array}{c}
 \begin{array}{l} \text{\textit{n}th} \\ \text{term} \end{array} \swarrow \\
 a_n = \underbrace{a_{n-1}}_{\substack{\text{previous} \\ \text{term}}} + d \\
 \begin{array}{l} \swarrow \\ \text{common} \\ \text{difference} \end{array}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{l} \text{\textit{n}th} \\ \text{term} \end{array} \swarrow \\
 g_n = \underbrace{g_{n-1}}_{\substack{\text{previous} \\ \text{term}}} \cdot r \\
 \begin{array}{l} \swarrow \\ \text{common} \\ \text{ratio} \end{array}
 \end{array}$$

Write the first five terms of the sequence.

1.  $a_1 = 5; a_n = a_{n-1} + 1$

$$a_1 = 5$$

$$a_2 = 5 + 1 = 6$$

$$a_3 = 6 + 1 = 7$$

$$a_4 = 7 + 1 = 8$$

$$a_5 = 8 + 1 = 9$$

2.  $a_2 = 36; a_n = \frac{1}{3}a_{n-1}$

$$a_2 = 36$$

$$a_3 = \frac{1}{3}(36) = 12$$

$$a_4 = \frac{1}{3}(12) = 4$$

$$a_5 = \frac{1}{3}(4) = \frac{4}{3}$$

$$a_6 = \frac{1}{3}\left(\frac{4}{3}\right) = \frac{4}{9}$$

Regents Exam Practice.

If a sequence is defined recursively by  $f(0) = 2$  and  $f(n + 1) = -2f(n) + 3$  for  $n \geq 0$ , then  $f(2)$  is equal to

- (1) 1  
(2) -11  
(3) 5  
(4) 17

$$f(0) = 2$$

$$f(1) = -2(2) + 3 = -1$$

$$f(2) = -2(-1) + 3 = 5$$

If  $f(1) = 3$  and  $f(n) = -2f(n - 1) + 1$ , then  $f(5) =$

- (1) -5  
(2) 11  
(3) 21  
(4) 43

$$f(1) = 3$$

$$f(2) = -2(3) + 1 = -5$$

$$f(3) = -2(-5) + 1 = 11$$

$$f(4) = -2(11) + 1 = -21$$

$$f(5) = -2(-21) + 1 = 43$$