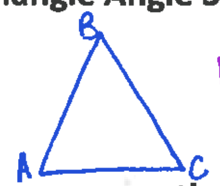
Geometry R – Mrs. Cowen
Unit 4 - Day 3 Notes

Name:_______________________
Date:_____________

Triangle Properties -
Triangle Angle Sum Theorem & Exterior Angle Theorem

Triangle Angle Sum Theorem: The angles of a triangle sum to 180 degrees.



Given: $\angle A \cong \angle C$ and $m\angle B$ is 10 more than 3 times $m\angle A$. Find $m\angle B$.

let $m\angle A = x$
$m\angle C = x$
$m\angle B = 3x + 10$

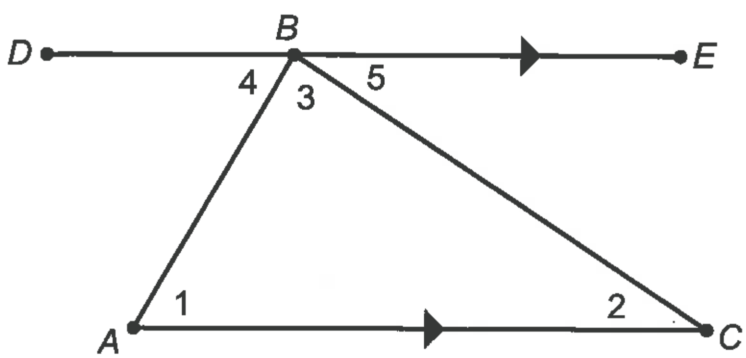
$x + x + 3x + 10 = 180$
$5x + 10 = 180$
$5x = 170$
$x = 34$

$m\angle B = 3(34) + 10$
$m\angle B = 112$

How can the Triangle Sum Theorem be proven true?

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180^o$



To prove the theorem true, we first draw an ***Auxiliary line*** ($\overline{DBE}$) parallel to one side of the triangle, as shown in the diagram.

a. How do we know that this parallel line even exists? Thru any point not on a line, there exists one line thru the point parallel to the given line

b. The auxiliary line makes $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 5$. Explain why.

When 2 parallel lines are cut by a transversal, alternate interior angles are congruent.

c. We also know $m\angle 4 + m\angle 3 + m\angle 5 = 180$. Explain why.

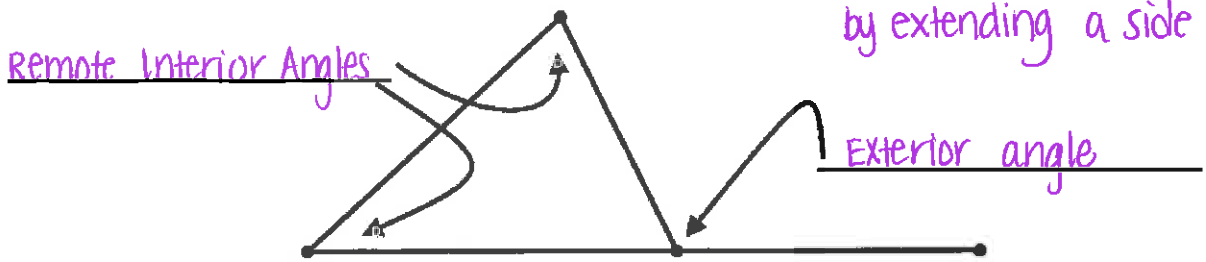
$\angle DBE$ is a straight angle so $m\angle DBE = 180$
By angle addition $m\angle 4 + m\angle 3 + m\angle 5 = 180$

d. Based on your answers from *part b* and *c*, explain why $m\angle 1 + m\angle 2 + m\angle 3 = 180$.

By substituting $\angle 1$ in for $\angle 4$ and $\angle 2$ in for $\angle 5$ into $m\angle 4 + m\angle 3 + m\angle 5 = 180$, we get $m\angle 1 + m\angle 2 + m\angle 3 = 180$ ☺

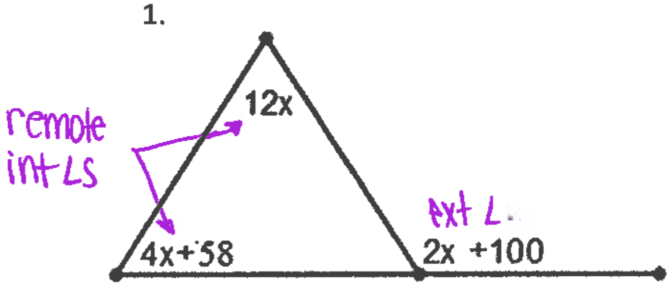
Exterior Angle Theorem: An **exterior angle** of a triangle is equal to the sum of the 2 **remote interior** angles of the triangle.

The exterior ∠ is formed by extending a side of the Δ.



Example: Find the value of the variables.

1.



$4x+58+12x = 2x+100$

$16x+58 = 2x+100$

$-2x \quad -2x$

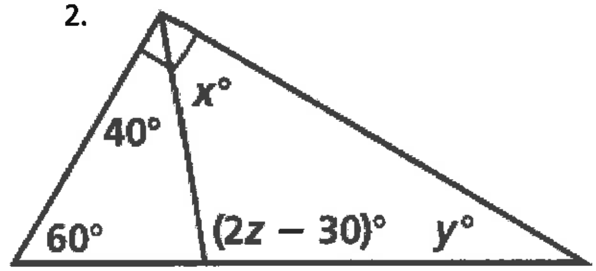
$14x+58 = 100$

$-58 \quad -58$

$14x = 42$

$x = 3$

2.



$40+60 = 2z-30$

$100 = 2z-30$

$+30 \quad +30$

$130 = 2z$

$\boxed{65 = z}$

$x+40=90$

$\boxed{x=50}$

$x+y+2z-30=180$

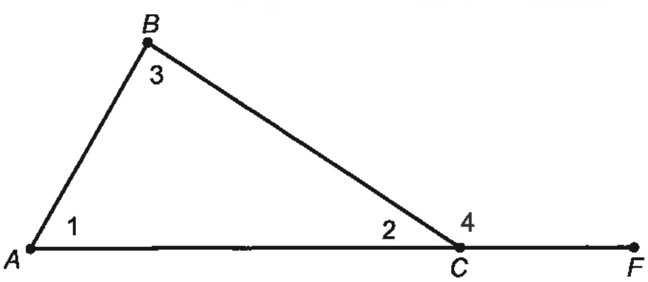
$50+y+2(65)-30=180$

$50+y+100=180$

$150+y=180$

$\boxed{y=30}$

How can the Exterior Angle Theorem be proven true?



To prove the theorem true, we can make use of the Triangle Angle Sum Theorem to justify that $m\angle 4 = m\angle 1 + m\angle 3$.

a. Explain why $m\angle 4 + m\angle 2 = 180$ and $m\angle 1 + m\angle 2 + m\angle 3 = 180$.

∠4 & ∠2 are supp

∠1, ∠2 & ∠3 are the ∠s of a Δ.

b. Explain why your results from *part a* make $m\angle 4 + m\angle 2 = m\angle 1 + m\angle 2 + m\angle 3$

by substitution

c. Explain why *part b* makes $m\angle 4 = m\angle 1 + m\angle 3$

if you subtract m∠2 from both sides you get m∠4 = m∠1 + m∠3