Geometry R
Unit 5 – Day 5 Notes v2

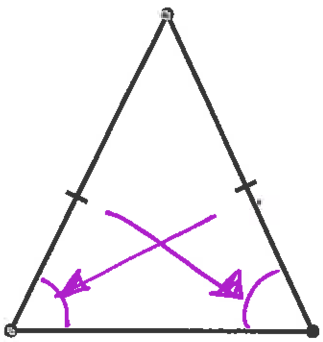
Name: ______________________
Date: ________________

Equilateral & Isosceles Triangles

Isosceles Triangle – A triangle with 2 congruent sides.

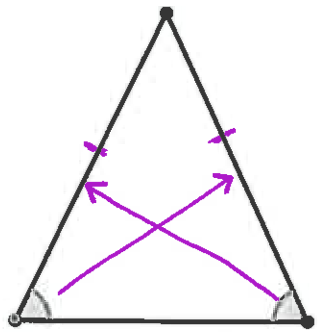
Isosceles Triangle Theorems:

Theorem 1: In a triangle, if 2 sides are congruent, then the angles opposite those sides are congruent.



In a Δ, if 2 sides $\cong$, the $\angle$s opp sides $\cong$.

Theorem 2: In a triangle, if 2 angles are congruent, then the sides opposite those angles are congruent.

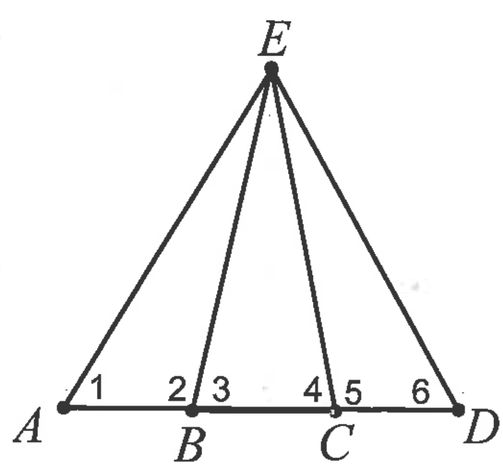


In a Δ, if 2 $\angle$s $\cong$, the sides opp $\angle$s $\cong$.

How the theorems Work:

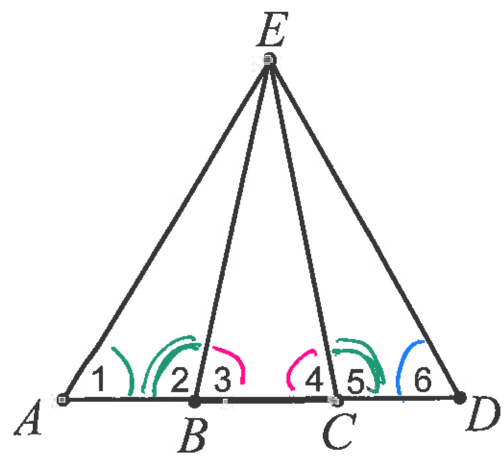
For each pair of **given congruent sides**, draw and label the Isosceles Triangle that contains those sides. Use **Theorem 1** to state the angles that must be congruent.

Congruent Sides	Triangle	Congruent Angles
$\overline{EB} \cong \overline{EC}$	ΔEBC	$\angle 3 \cong \angle 4$
$\overline{EA} \cong \overline{ED}$	ΔEAD	$\angle 1 \cong \angle 6$
$\overline{EA} \cong \overline{EC}$	ΔEAC	$\angle 1 \cong \angle 4$



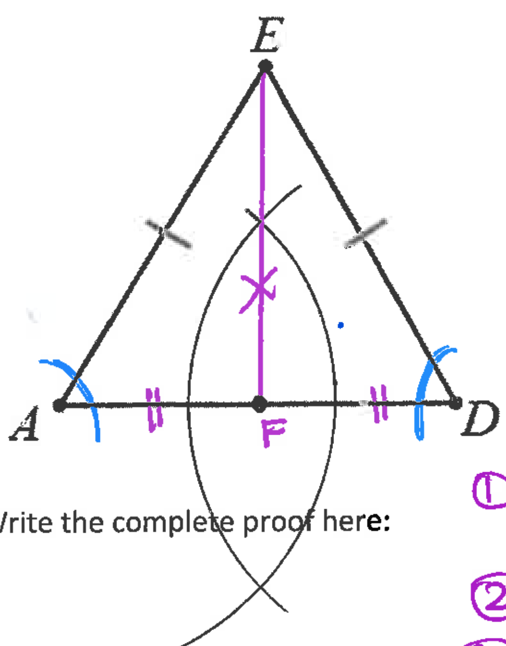
For each pair of **given congruent Angles**, draw and label the Isosceles Triangle that contains those sides (if possible). Use **Theorem 2** to state the sides that must be congruent.

Congruent Angles	Triangle	Congruent Sides
$\angle 3 \cong \angle 4$	$\triangle EBC$	$\overline{EB} \cong \overline{EC}$
$\angle 1 \cong \angle 6$	$\triangle EAD$	$\overline{EA} \cong \overline{ED}$
$\angle 2 \cong \angle 5$	$\triangle EBC$	*Need supp $\angle$s $\angle 2$ supp $\angle 3$ $\angle 5$ supp $\angle 4$ > $\angle 3 \cong \angle 4$ $\overline{EB} \cong \overline{EC}$



Why are the theorems true?

4. We can prove the theorems are true by making use of our triangle congruency postulates.



For example, for **Theorem 1** we have:

Given: Isosceles Triangle AED ($\overline{AE} \cong \overline{DE}$)
Prove: The angles opposite are congruent ($\angle A \cong \angle D$)

1. Use a compass and straight edge to construct the midpoint of side $\overline{AD}$.

2. Use the two triangles to justify that $\angle A \cong \angle D$.

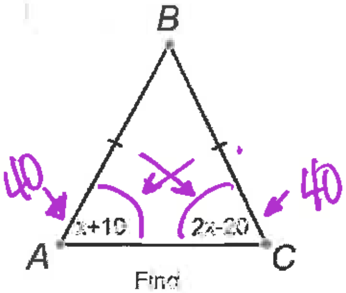
*(Theorem **2** can be proven true in a similar way.)*

Write the complete proof here:

① $\overline{AE} \cong \overline{DE}$ F is midpt $\overline{AD}$	① Given
② $\overline{AF} \cong \overline{FD}$	② midpt ÷ seg into 2 $\cong$ seg
③ $\overline{EF} \cong \overline{EF}$	③ reflexive
④ $\triangle AEF \cong \triangle DEF$	④ SSS
⑤ $\angle A \cong \angle D$	⑤ corr parts $\cong \triangle \cong$

Examples Using the Theorems:

1.



Find $m\angle B$

$40 + 40 = 80$ $\quad$ $180 - 80 = 100 = m\angle B$

*Pictures Not drawn to scale!!

$x+10 = 2x-20$

$-x \qquad -x$

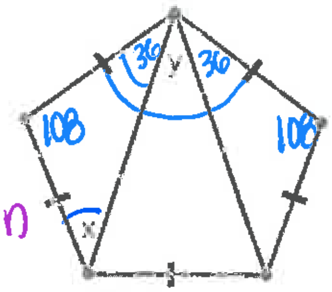
$10 = x-20$

$+20 \qquad +20$

$30 = x$

$\angle B = 100$

2.



Find x and y

$(5-2)(180) = 540$

$\frac{540}{5} = 108$

Hint: How big is each angle of the regular pentagon?

$2x+108=180$

$2x = 72$

$x = 36°$

$108 = 2(36)+y$

$108 = 72+y$

$36° = y$

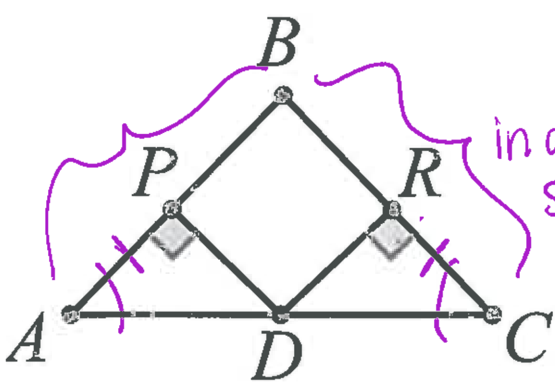
3. Complete the proof:

Given: $\overline{AB} \cong \overline{CB}$

$\overline{AP} \cong \overline{CR}$

$\angle CRD$ & $\angle APD$ are right $\angle$'s $\angle$

Prove: $\Delta APD \cong \Delta CRD$



$\boxed{\overline{AB} \cong \overline{CB}}$ Given

$\downarrow$

$\boxed{\angle A \cong \angle C}$ in a Δ if 2 $\angle$s $\cong$ sides opp $\angle$s are $\cong$

$\boxed{\overline{AP} \cong \overline{CR}}$ Given

$\boxed{\angle CRD \text{ \& } \angle APD \text{ rt } \angle s}$ Given

$\downarrow$

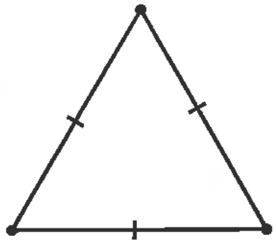
$\boxed{\angle CRD \cong \angle APD}$ all rt $\angle$s $\cong$

$\boxed{\Delta APD \cong \Delta CRD}$

ASA

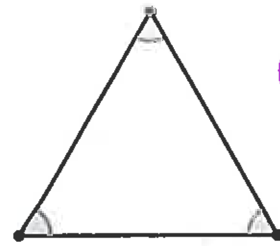
sides

Equilateral Triangle: A triangle with 3 congruent Sides.



angles

Equiangular Triangle: A triangle with 3 congruent angles.

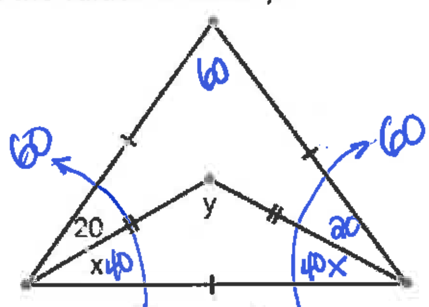


all ∠s are 60°

a. What is the measure of each angle of an equilateral triangle? Explain.

60° an equilateral Δ has 3 equal sides, making 3 = angles. A Δ has 180° so 180 ÷ 3 = 60°

b. Find the values of x and y.



Find x and y

$x + 20 = 60$

$x = 40$

$$\begin{array}{r} 40 \\ +40 \\ \hline 80 \end{array} \qquad \begin{array}{r} 180 \\ -80 \\ \hline 100 \end{array} = y$$