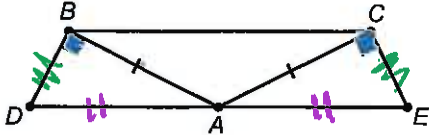


## Overlapping Triangles & HL Proofs

Given: Isosceles Triangle  $ABC$  •  
A is a midpoint of  $\overline{DE}$   
 $\angle DBA$  and  $\angle ECA$  are rt  $\angle$ 's

Prove:  $\overline{BD} \cong \overline{CE}$



1. After assessing the problem situation, a student quickly decides they are going to use vertical angles to prove  $\triangle ABD \cong \triangle ACE$  by A.S.A. Explain how the student has incorrectly assessed the situation.

There are NO vertical  $\angle$ 's.

2. Complete a proof of the problem above that uses the HL congruency postulate.

① ISOS  $\triangle ABC$  •  
A is midpt  $\overline{DE}$  •  
 $\angle DBA$  &  $\angle ECA$  rt  $\angle$ 's •

②  $\overline{AB} \cong \overline{AC}$

③  $\overline{AD} \cong \overline{AE}$

④  $\angle DBA \cong \angle ECA$

⑤  $\triangle DBA \cong \triangle ECA$

⑥  $\overline{BD} \cong \overline{CE}$

① Given

② An isos  $\triangle$  has 2  $\cong$  sides

③ midpt  $\div$  seg into 2  $\cong$  seg

④ all rt  $\angle$ 's  $\cong$

⑤ HL

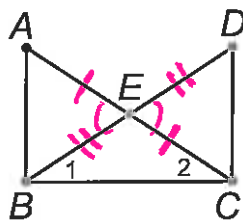
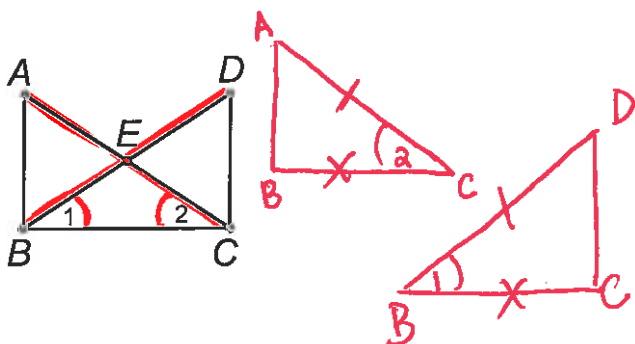
⑥ corr parts  $\cong$   $\triangle$   $\cong$

**Overlapping Triangles** – Often times a geometry picture will contain many overlapping triangles and it is unclear which triangles should be shown congruent. In these cases, redrawing the picture to single out the suspected congruent triangles can help determine a direction for the proof.

**Example:** For each problem, redraw the picture to separate out the suspected congruent triangles based on the given information. *Hint: Which triangles contain the parts in the "given" statements? Name the two triangles drawn and complete a proof.*

3a. Given:  $\angle 1 \cong \angle 2$   
 $\overline{AC} \cong \overline{DB}$   
 Prove:  $\overline{AB} \cong \overline{DC}$

3b. Given:  $\overline{AC}$  &  $\overline{BD}$  bisect each other  
 Prove:  $\overline{AB} \cong \overline{DC}$



- ①  $\angle 1 \cong \angle 2$
  - ②  $\overline{BC} \cong \overline{BC}$
  - ③  $\triangle ABC \cong \triangle DCB$
  - ④  $\overline{AB} \cong \overline{DC}$
- ① given
  - ② reflexive
  - ③ SAS
  - ④ corr parts  $\cong \triangle$

$\overline{AC}$  &  $\overline{BD}$  bisect each other

Given

↓

$E$  midpt  $\overline{AC}$  &  $\overline{BD}$

seg bis passes thru midpt

↓

$\overline{AE} \cong \overline{EC}$  ;  $\overline{BE} \cong \overline{ED}$

midpt  $\div$  seg into 2  $\cong$  seg.

↓

$\angle AEB \cong \angle CED$

vertical  $\angle$   $\cong$ .

↓

$\triangle AEB \cong \triangle CED$

SAS

↓

$\overline{AB} \cong \overline{DC}$

corr parts  $\cong \triangle$

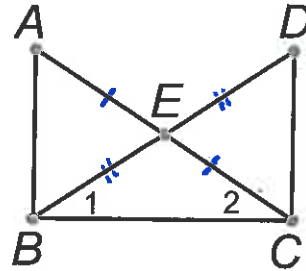
## Proving triangles congruent using parts from other congruent triangles

Sometimes a proof problem may require you to prove two triangles congruent based off parts from other congruent triangles. Essentially, this requires you to prove more than one pair of triangles congruent in a single proof.

Consider this example:

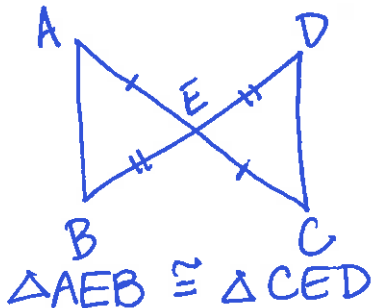
Given:  $\overline{AC}$  &  $\overline{BD}$  bisect each other  
 $\overline{AC} \cong \overline{DB}$

Prove:  $\angle 1 \cong \angle 2$

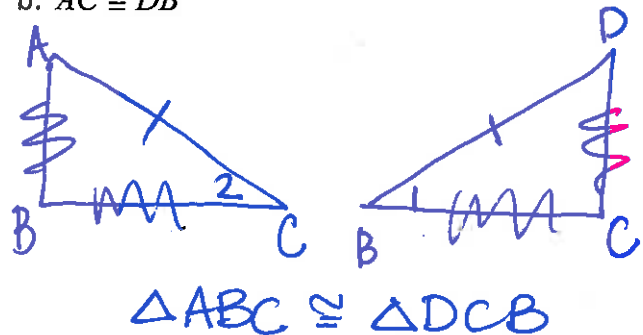


4. For each "given" statement, redraw the picture to separate out the suspected congruent triangles and name the two triangles drawn.

a.  $\overline{AC}$  &  $\overline{BD}$  bisect each other



b.  $\overline{AC} \cong \overline{DB}$



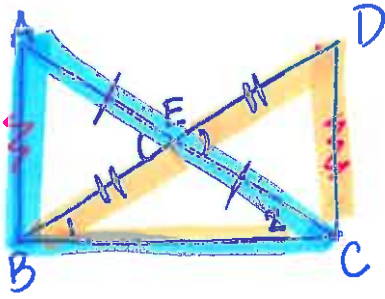
5. Of the two pairs of triangles drawn and named in #4, which pair does  $\angle 1$  and  $\angle 2$  belong to?

$\triangle ABC \cong \triangle DCB$

6. Based on your drawings in #4, how will showing  $\triangle AEB \cong \triangle CED$  help in showing  $\triangle ACB \cong \triangle DBC$ ? Explain.

- It will give us  $\overline{AB} \cong \overline{CD}$  by corr parts  $\cong \triangle \cong$
- We also need  $\overline{BC} \cong \overline{BC}$  by reflexive
- $\triangle ABC \cong \triangle DCB$  by SSS

7. Complete a proof for this problem on the back of this page.



①  $\overline{AC} \cap \overline{BD}$  bisect each other  
 $\overline{AC} \cong \overline{DB}$

② E is midpt of  $\overline{AC} \cap \overline{BD}$

③  $\overline{AE} \cong \overline{EC}$  ;  $\overline{BE} \cong \overline{ED}$

④  $\angle AEB \cong \angle CED$

⑤  $\triangle AEB \cong \triangle CED$

⑥  $\overline{AB} \cong \overline{DC}$

⑦  $\overline{BC} \cong \overline{BC}$

⑧  $\triangle ABC \cong \triangle DCB$

⑨  $\angle 1 \cong \angle 2$

① given

② seg bis passes thru midpt

③ midpt  $\div$  seg into 2  $\cong$  seg

④ vertical  $\angle$ s  $\cong$

⑤ SAS

⑥ corr parts  $\cong \triangle \cong$

⑦ reflexive

⑧ SSS

⑨ corr parts  $\cong \triangle \cong$